CONVECTION EXCITED BY A HIGH-FREQUENCY ELECTRIC CURRENT

A. V. Lykov, B. M. Berkovskii,
and Wu Zui Kwang

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The intensity and structure of gravitational convection of a conducting fluid in a vertical channel excited by a high-frequency current are studied. It is shown that no dynamic boundary layer exists. Values of the parameters Bi and n are found up to which it is meaningful to take convection into account.

1. Let us examine the question of exciting convection by a high-frequency current in a heavy electrical conducting fluid. The Joulean heat liberated as direct or alternating current flows will form an inhomogeneous temperature field in the fluid. In a gravitational field this will result in the appearance of Archimedes forces which cause convection in the majority of cases. The intensity and structure of the thermal gravitational convection should depend essentially on the frequency of the applied electrical field. In fact, in contrast to direct current, a rapidly alternating current is distributed nonuniformly over the cross section of a homogeneous conductor. It is concentrated on the surface (skin effect) [1]. This phenomenon implies a redistribution of the heat liberation and temperature in a liquid conductor, which is felt in turn by the excited convection and the convective heat exchange. Since the electrical and thermophysical parameters of conductors generally depend on temperature, then the thermoconvective phenomena can exert a reverse effect on the distribution of the current density over the cross section.

The question of convection excited by a high-frequency current and its influence on heat exchange in conducting media has hardly been studied. Several experimental papers [2, 3] can be mentioned, where [2] is devoted to an investigation of boiling heat exchange under conditions of direct heating of the heat-exchange surface by high-frequency currents. In this paper the boiling heat-exchange coefficients were measured for benzene, acetone, ethanol in copper, brass, and chromed copper tubes placed in a stainless steel cylindrical vessel. The influence of a high-frequency electromagnetic field on heat exchange with free convection of a fluid in a rectangular steel vessel was investigated in [3]. The experiments were conducted with water and benzene in heaters of the same materials as in [2]. The measurements carried out showed that the high-frequency electromagnetic field intensifies heat exchange with free convection. The intensifying effect of the field is determined by its intensity, and the nature and temperature of the fluid being heated. The paper [4] is devoted to temperature distribution in a plane conducting layer. A brief analysis of the dimensionless temperature profile is given therein.

2. Let us formulate the mathematical problem of thermal gravitational convection in an electrically conductive fluid because of Joulean heat liberation from a high-frequency electrical current. Let us start from the Maxwell and continuity equations and the energy and momentum conservation laws. Let us make a number of simplifying assumptions. Let us assume that the heat liberation is not very large and that the thermoconvective part of the problem can be written down in the Boussinesq approximation [5]:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\,\mathbf{v} = -\frac{1}{\rho}\nabla \mathbf{p} + \nu\nabla^2\mathbf{v} - \mathbf{k}g\beta\,(T - T^*),$$

$$\operatorname{div}\mathbf{v} = 0,$$

$$\rho c_p\left[\frac{\partial T}{\partial t} + (\mathbf{v}\cdot\nabla)\,T\right] = \lambda\nabla^2 T + \frac{j^2}{\sigma}.$$

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TABLE 1. Values of the Parameter n

l, m	$\omega:2\pi$			n
	mercury	melted gallium	melted silver	
$5\cdot10^{-2}$	$0{,}953\cdot10^{2}$	$0{,}264\cdot10^{2}$	$0{,}198\cdot10^{2}$	1
	$0{,}238\cdot10^{4}$	$0{,}658\cdot10^{3}$	$0{,}46\cdot10^{3}$	5
	$0{,}91\cdot10^{4}$	$0{,}264\cdot10^{4}$	$0{,}183\cdot10^{4}$	10
$5\cdot10^{-3}$	$0{,}953\cdot10^{4}$	$0{,}264\cdot10^{4}$	$0{,}198\cdot10^{4}$	1
	$0{,}238\cdot10^{6}$	$0{,}658\cdot10^{5}$	$0{,}46\cdot10^{5}$	5
	$0{,}91\cdot10^{6}$	$0{,}264\cdot10^{6}$	$0{,}183\cdot10^{6}$	10

Let us consider the displacement current density to be considerably less than the conduction currents, and the period of field variation to be large compared to the mean free path time of the charge carriers in the conductor. Then, neglecting the volume charges, the electromagnetic field equations become [1]:

$$\operatorname{div}\mathbf{B}=0,\ \operatorname{div}\mathbf{D}=0,$$
$$\mathbf{B}=\mu\mathbf{H},\ \mathbf{D}=\varepsilon\mathbf{E},$$
$$\operatorname{rot}\mathbf{H}=\frac{4\pi}{c}\,\mathbf{j},\ \operatorname{rot}\mathbf{E}=-\frac{1}{c}\cdot\frac{\partial\mathbf{B}}{\partial t},$$
$$\mathbf{j}=\sigma\mathbf{E}. \tag{1}$$

If the change in the conductivity σ and permittivity μ in the volume under consideration is neglected, then it is easy to obtain the following equation to determine the electrical field within the liquid conductor:

$$\nabla^2\mathbf{E}=\frac{4\pi\mu\sigma}{c^2}\cdot\frac{\partial\mathbf{E}}{\partial t}. \tag{2}$$

An analogous equation is obtained for the magnetic field.

We limit ourselves herein to the examination of the monochromatic fields $\mathbf{E}=\mathbf{E}_0(r)\exp(i\omega t)$, for which (2) goes over into the following equation in the amplitude of the electrical vector:

$$\nabla^2\mathbf{E}_0=\frac{4\pi\mu\sigma}{c^2}\,i\omega\mathbf{E}_0. \tag{2'}$$

Together with the appropriate boundary conditions, the system (1), (2') determines the thermoconvective processes in a high-frequency electrical field in the approximation chosen.

3. Let us study the one-dimensional problem in a vertical channel with solid nonconducting, impermeable walls in detail. Let a monochromatic high-frequency current be passed along the channel. In this case, the system of equations

$$\frac{\partial^2 E_y}{\partial x^2}=\frac{4\pi\sigma\omega}{c^2}\,iE_y,$$
$$\rho c_p\frac{\partial T}{\partial t}=\lambda\frac{\partial^2 T}{\partial x^2}+\frac{j_y^2}{\sigma},\quad \nu\frac{\partial^2 v}{\partial x^2}+\beta g(T-T^*)=0 \tag{3}$$

follows from (1)-(2').

Let us consider time segments much greater than the period of current fluctuation. Then, the mean Joulean heat liberation per period

$$\left\langle\frac{j_y^2}{\sigma}\right\rangle=\frac{\sigma}{\tau}\int_0^\tau E_y^2dt=E_y^2(x)\,\frac{\sigma}{\tau}\int_0^\tau\exp(2\omega it)\,dt=\sigma\,\frac{|E_y(x)|^2}{2}\,(x)$$

can be used in the energy equation, and it can be considered that the temperature is independent of the time. After this, the system (3) will be in dimensionless form

$$\frac{\partial^2 E_1}{\partial x_1^2}=2in^2E_1,$$
$$\frac{\partial^2\theta}{\partial x_1^2}=-|E_1|^2, \tag{3'}$$
$$\frac{\partial^2 v_1}{\partial x_1^2}=-\theta,$$

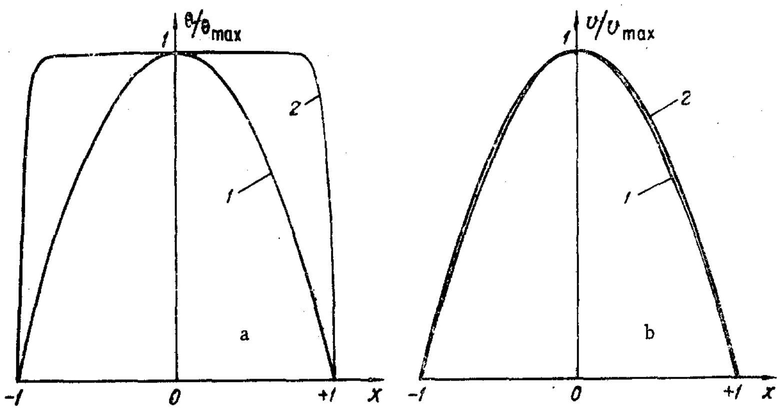

Fig. 1. Relative temperature (a) and relative velocity (b) distributions: 1) n = 1; 2) 15.

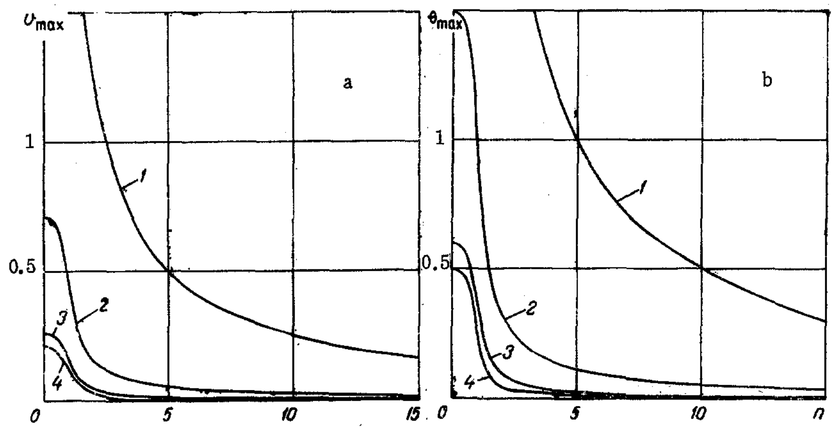

Fig. 2. Maximal velocities (a) and temperatures (b) as a function of n for different Bi: 1) Bi = 0.1; 2) 1; 3) 10; 4) ∞.

with the boundary conditions for the field and the velocity

$$E_1(\pm 1) = 1,\ v_1(\pm 1) = 0. \quad (4)$$

Let us consider boundary conditions of the third kind for the temperature in this paper [6]:

$$\frac{\partial\theta}{\partial x_1} = -\mathrm{Bi}\,\theta \text{ for } x_1 = +1,\quad \frac{\partial\theta}{\partial x_1} = 0 \text{ for } x_1 = 0, \quad (4)$$

where the following dimensionless quantities have been introduced:

$$E_y = E_1 E_0,\ x = x_1 l,\ v = v_1 V,\ T - T_\infty = \theta\Delta T,$$

$$T_\infty = T^*,\ \Delta T = \frac{\sigma l^2 E_0^2}{2\lambda},\ V = \frac{\beta g l^2}{\nu}\Delta T$$

and the parameter $n = l\sqrt{2\pi\sigma\omega}/c$ characterizes the ratio of the channel half-width l to the thickness of the electrical skin-layer $\delta = c/\sqrt{2\pi\sigma\omega}$. It is proportional to the channel half-width l, the circular frequency ω, and the conductivity σ. Typical values of the parameter n are presented in Table 1 for mercury, melted gallium, and silver.

Let us write down the solution of the system (3') with the boundary conditions (4) (the subscript 1 is henceforth omitted)

$$|E| = \sqrt{\frac{\operatorname{ch} 2nx + \cos 2nx}{\operatorname{ch} 2n + \cos 2n}},$$

$$\theta = \frac{N_2(2n) - N_2(2nx)}{4n^2 N_1(2n)} + \frac{1}{\mathrm{Bi}} \cdot \frac{\operatorname{sh} 2n + \sin 2n}{2nN_1(2n)}, \quad (5)$$

$$v = \frac{1}{16n^4 N_1(2n)}\left[2n^2 N_2(2n) - N_1(2n) - 2n^2x^2 N_2(2n) + N_1(2nx)\right] + \frac{1}{\mathrm{Bi}} \cdot \frac{\operatorname{sh} 2n + \sin 2n}{4nN_1(2n)}(1 - x^2),$$

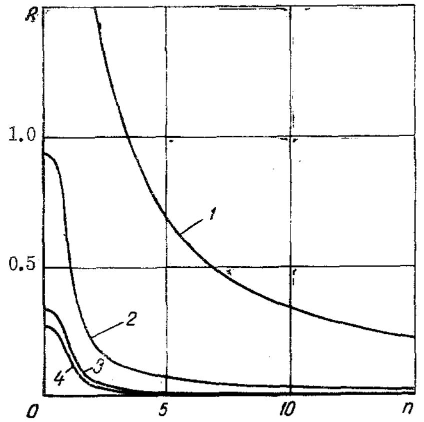

Fig. 3

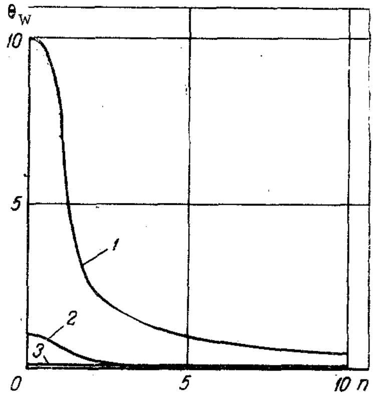

Fig. 4

Fig. 3. Fluid discharge through a channel as a function of n for different Bi: 1) Bi = 0.1; 2) 1; 3) 10; 4) ∞.

Fig. 4. Wall temperatures as a function of n for different Bi: 1) Bi = 0.1; 2) 1; 3) 10.

$$N_{1,2}(\xi)=\operatorname{ch}\xi\pm\cos\xi.$$

Therefore, the temperature and velocity depend on the two parameters n, Bi and are symmetric relative to the center of the channel. Let us compute the fluid discharge through the channel:

$$R=\frac{1}{16n^4N_1(2n)}\left[\frac{8n^2}{3}N_2(2n)-2N_1(2n)+\frac{1}{n}(\operatorname{sh}2n+\sin 2n)\right]+\frac{\operatorname{sh}2n+\sin 2n}{\mathrm{Bi}\,3nN_1(2n)}. \tag{6}$$

The heat fluxes on the walls are identical in magnitude and equal

$$Q=\frac{\operatorname{sh}2n+\sin 2n}{2nN_1(2n)}. \tag{7}$$

The temperatures on the walls are

$$\theta_w=\frac{1}{\mathrm{Bi}}\,\frac{\operatorname{sh}2n+\sin 2n}{2nN_1(2n)}.$$

Let us analyze the velocity and temperature distribution, and the discharge and heat fluxes in more detail. For a fixed value of Bi the convection intensity is a maximum when the parameter $n\to 0$. In the limit case $n = 0$, formula (5) simplifies considerably

$$\theta=\frac{1-x^2}{2}+\frac{1}{\mathrm{Bi}},\quad v=\frac{1-x^2}{4}+\frac{1}{2\mathrm{Bi}}(1-x^2).$$

In the other limit case $n \gg 1$, the temperature is almost constant everywhere except at the boundary layers near the walls where it changes abruptly. This phenomenon is similar to the electrical skin effect. As $n\to\infty$ the velocity and temperature profiles depend still less on Bi and tend to zero. For this there are two reasons. Firstly, for large n, i.e., as the frequency rises, the fundamental Joulean heat sources are concentrated near the walls, hence, the heat is easily eliminated through the walls. Secondly, as $n\to\infty$ the thickness of the layer conducting the current diminishes and the resistance of the conductor should increase, whereupon the dc current should become less. This also implies a diminution in the heat liberation. In practice, this all holds even for $n \simeq 15$.

An analysis of the expression for the velocity (5) showed that, in contrast to the thermal boundary layer, no dynamic boundary layer exists. The relative temperature distribution θ/θ_{max} and the velocity distribution v/v_{max} are constructed for Bi = ∞ in Figs. 1a and b. It follows from the graph that v/v_{max} has almost the identical form for large and small n while a quite definite boundary layer appears for θ/θ_{max} as n grows.

The maximal temperature θ_{max} and velocity v_{max} are shown in Fig. 2a, b as a function of n for different Bi. For small values of n the θ_{max} and v_{max} increase rapidly, especially for small Bi (up to Bi = 1). The smaller the Bi, the more rapidly they increase. As n grows θ_{max} and v_{max} tend to zero. An analogous picture holds for the discharge R and the wall temperature θ_w (Figs. 3 and 4). The heat flux is independent of Bi and has a maximum value equal to one for $n = 0$. It drops sharply for small n and then tends to zero for large n.

TABLE 2. Maximal Velocity and Temperature

ω, Hz	Mercury		Water		Mercury (for natural convection)	Water (for natural convection)
	v_{max}	T_{max}+293 °K	v_{max}, m/sec	T_{max}+293 °K	v_{max}	
0	$7{,}96\cdot10^{-2}$	0,5	$1{,}35\cdot10^{-2}$	7,5	$3\cdot10^{-2}$	$3\cdot10^{-3}$
10^5	$3{,}68\cdot10^{-2}$	0,266	$6{,}15\cdot10^{-3}$	3,4		
10^7	$2{,}2\cdot10^{-4}$	$0{,}111\cdot10^{-2}$	$3{,}74\cdot10^{-5}$	$1{,}66\cdot10^{-2}$		

Let us examine the particular case when Bi → ∞ (in practice Bi ≃ 100), i.e., the heat exchange between the walls and the surrounding medium occurs by the Newton law with a constant temperature, but with an infinite heat-exchange coefficient α. Then the wall temperatures are identical and we obtain all the expressions for θ, v, R without terms containing Bi. The curves for Bi = ∞ correspond to this case in the graphs.

It should be noted that the results obtained are valid for Bi arbitrarily small but not equal to zero since the walls are heat insulated for Bi = 0 and the process cannot be stationary.

An analysis of (5)-(7) and Figs. 2-4 shows that the convection intensity is substantial for Bi ≥ 1 up to n ≤ 5, and in the range 0, 1 ≤ Bi < 1 up to n ≤ 15. For a more exact quantitative comparison of the convection excited by a high-frequency current, the dependence between the parameters Bi and n was found for convection with direct current, for which the velocity was 1/10 the dc velocity. This dependence can be approximated by the equation n = 0.8/Bi + 2.8 for 0.084 ≤ Bi ≤ 10 and n = 2.8 for Bi ≥ 10.

Let us present numerical estimates showing to what frequencies it is meaningful to take account of the convection excited by a high-frequency current.

Presented in Table 2 are values of the maximal velocity v_{max} and temperature T_{max} for Bi = ∞ for mercury and water if it is considered that its conductivity is the same as for mercury. Also presented for comparison is the value of v_{max} for natural convection. In this latter case, as the temperature drops one degree, the v_{max} of water has the order of mm/sec while for mercury it is tenfold greater. In the case of convection caused by a high-frequency current with a frequency on the order of 10^5 Hz, the maximal velocities are of the same order.

NOTATION

σ, λ, ν	are the electrical conductivity, heat conductivity, and kinematic viscosity coefficients;
β, α	are the coefficients of thermal expansion of a fluid and heat exchange;
ω	is the circular current frequency;
c	is the speed of light;
g	is the acceleration of gravity;
i	is the imaginary unit;
x	is the coordinate perpendicular to the channel;
E	is the electrical field intensity;
T	is the fluid temperature;
v	is the fluid velocity;
l	is the channel half-width;
$Bi = \alpha l/\lambda$	is the Biot criterion.

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